

# Performance and Stability of Schottky Barrier Mixers

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(Manuscript received June 21, 1972)

*We discuss the performance of a Schottky barrier diode as a mixer when the barrier of the diode is open-circuited at the harmonics  $2\omega_o$ ,  $3\omega_o$ , etc. of the pump frequency  $\omega_o$ . Such a mixer is shown to be capable of arbitrarily high conversion gain provided*

$$\omega_c \geq \eta \omega_o,$$

*where  $\omega_c$  is the cutoff frequency of the diode and  $\eta$  is a parameter that is typically less than 6.25 and approaches 4 under certain ideal conditions. It is shown that the limit imposed by the series resistance of the diode on the double-sideband noise figure of the mixer is given by*

$$F_m > \left(1 - \eta \frac{\omega_o}{\omega_c}\right)^{-1}.$$

*An experiment is described at 1.25 GHz on a room temperature mixer whose double-sideband noise figure  $F_m$  as a function of gain has a minimum of about 0.7 dB (for gain less than unity) and a maximum of about 2.3 dB (for high gain).*

## I. INTRODUCTION

In the past five years the performance of microwave mixers has been substantially improved with the advent of high quality Schottky barrier diodes.<sup>1</sup> The noise figures obtained so far are better by a factor of approximately 2 than those obtained previously using point-contact diodes.<sup>1-4</sup> The ultimate microwave noise figure obtainable with these devices is not yet known, but there is reason to believe that, at room temperature, a figure well under 3 dB is possible.

Calculation in a previous article<sup>5</sup> showed that a Schottky barrier diode with suitable characteristics should have a noise figure well under 1 dB provided the barrier of the diode is open-circuited at the harmonics

$2\omega_o$ ,  $3\omega_o$ , etc. of the pump frequency  $\omega_o$ . However, that calculation neglects the barrier capacitance and is therefore valid only at low frequency. The present purpose is to investigate the effect of the barrier capacitance. Three main assumptions are made: (i) the barrier of the diode is open-circuited at  $2\omega_o$ ,  $3\omega_o$ , etc., (ii) the output frequency  $p$  of the mixer is very low with respect to  $\omega_o$ , and (iii) the diode can be represented by an equivalent circuit discussed in Section II.

A mixer is commonly considered to be a linear transducer having finite maximum gain and a noise temperature ratio close to unity; the maximum gain is usually considered its most important attribute.<sup>1</sup> However, this picture is not valid in general for the mixer under consideration here. It will be shown that this mixer is potentially unstable if the cutoff frequency  $\omega_c$  of the diode is sufficiently high with respect to  $\omega_o$ . Thus its gain is unlimited, in the sense that it can be made arbitrarily high by appropriately choosing the terminations at the input, image, and output frequencies (i.e., at  $\omega_o \pm p$  and  $p$ ).

In a previous article<sup>2</sup> the mechanism responsible for instability in a mixer was discussed and necessary and sufficient conditions for unconditional stability were derived. These conditions, given in Section II, are used to determine the relation between mixer stability and mixer parameters. The main result is that a mixer is potentially unstable (i.e., that high gain is possible) if and only if

$$\omega_c \geq \omega_o \eta, \quad (1)$$

where  $\eta$  is a parameter the value of which depends primarily on the breakdown voltage  $V_B$  of the diode. It is shown that  $\eta \rightarrow 4$  as  $V_B \rightarrow \infty$  and that typically

$$4 < \eta < 6.25. \quad (2)$$

The value  $\eta$  has important significance in connection with the noise performance of a mixer at high gain because the limit imposed by the series resistance of the diode on the ultimate noise performance is given by the inequality

$$F_m > \left(1 - \eta \frac{\omega_o}{\omega_c}\right)^{-1}, \quad (3)$$

where  $F_m$  is the double-sideband noise figure.

Following the analysis of Ref. 5, an experiment was undertaken to determine the performance obtainable from a mixer satisfying assumptions (i), (ii), and (iii). We designed such a mixer and measured its

behavior as discussed in Section VI. It was, as expected, potentially unstable. The double-sideband noise figure as a function of gain was found to have a minimum value of 0.7 dB, occurring at a gain less than unity, and a maximum of about 2.3 dB, at high gain.

High gain in a mixer is no new phenomenon; it was demonstrated both theoretically and experimentally more than 20 years ago.<sup>7</sup> Since then, the effect of the barrier capacitance has been treated by several authors.<sup>8-12</sup> However, to the best of our knowledge, the effect of the barrier capacitance in a mixer satisfying assumption (i) has never been studied before. The amplifying ability is not a surprising property (Ref. 6), but good noise performance at high gain is perhaps unexpected.

## II. PRELIMINARY CONSIDERATIONS

The equivalent circuit of Fig. 1 is assumed for the Schottky barrier diode; it consists of a small series resistance  $R_s$  and two nonlinear elements, the barrier capacitance  $C(v_b)$  and the barrier resistance  $R(v_b)$ . The capacitance  $C(v_b)$  and the current  $i_R$  through  $R(v_b)$  are assumed to obey the familiar relations

$$C(v_b) = \frac{C_0 \sqrt{\phi}}{\sqrt{\phi - v_b}} \quad (4)$$

and

$$i_R = i_s \left[ \exp \left( \frac{qv_b}{kT} \right) - 1 \right], \quad (5)$$

where  $\phi$  is the contact potential,  $i_s$  the saturation current,  $q$  the electronic charge,  $k$  the Boltzman constant, and  $T$  the absolute temperature;  $q/kT \cong 40$  for  $T \cong 290^\circ\text{K}$ .

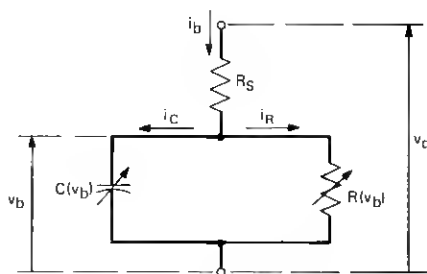


Fig. 1—Schottky barrier diode.

Figure 2 shows a two-terminal-pair network  $M$  driven by a sinusoidal current

$$i(t) = 2I \cos \omega_o t \quad (6)$$

and a direct current  $I_o$ . This network consists of the diode and two filters  $F_o$  and  $F_{\omega_o}$ . According to assumption (ii), we assume for  $F_o$  and  $F_{\omega_o}$  the following characteristics at the harmonics of  $\omega_o$  (and in their vicinity):  $F_o$  is a short circuit at dc and an open circuit at  $\omega_o$ ,  $2\omega_o$ ,  $3\omega_o$ , etc.;  $F_{\omega_o}$  is a short circuit at  $\omega_o$  and an open circuit at dc,  $2\omega_o$ ,  $3\omega_o$ , etc. From Fig. 2 the terminal current of the diode is

$$i_b(t) = I_o + i(t). \quad (7)$$

The voltage  $v_b$  across the barrier is assumed periodic with frequency  $\omega_o$ ,

$$v_b(t) = v_{bo} + 2 \operatorname{Re} (V_{b1} e^{j\omega_o t} + \dots), \quad (8)$$

where the dots indicate components at  $2\omega_o$ ,  $3\omega_o$ , etc. Let  $Z_b$  denote the impedance presented by the barrier at  $\omega_o$ .

$$Z_b = R_b + jX_b = \frac{V_{b1}}{I}. \quad (9)$$

Then the impedance  $Z$  presented by the network at  $\omega_o$  is

$$Z = R + jX = R_s + Z_b. \quad (10)$$

The terminal voltage at dc is

$$V_o = V_{bo} + R_s I_o. \quad (11)$$

When this network (designated here by  $M$ ; see Fig. 2) is used as a

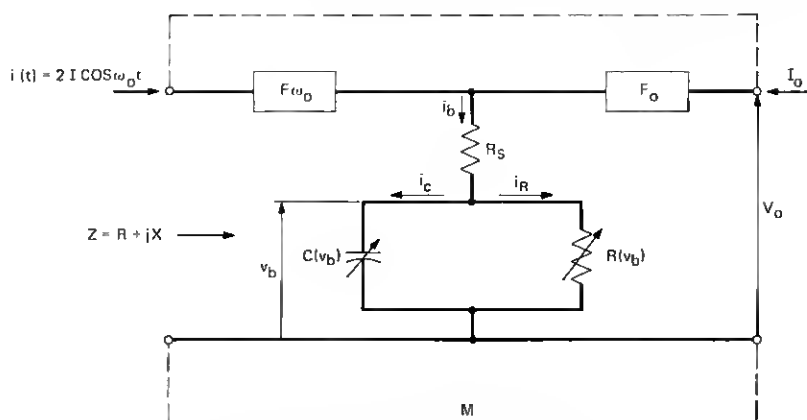


Fig. 2—Network  $M$  representing the mixer.

mixer, a small signal is applied at the input frequency  $\omega_o + p$  (or  $\omega_o - p$ ) and suitable terminations are provided at the image and output frequencies  $\omega_o - p$  (or  $\omega_o + p$ ) and  $p$ . Signal power then flows out of the network at  $\omega = p$  to the output termination. The conversion gain, which is defined\* here as the ratio of the output power to the power available from the input signal generator, has a finite maximum value only if  $M$  is unconditionally stable. If  $M$  is *not* unconditionally stable (i.e., if  $M$  is potentially unstable), its gain can be made arbitrarily high by properly choosing the terminations at  $\omega_o \pm p$  and  $p$ . To determine the conditions for which  $M$  is potentially unstable, we assume  $p \ll \omega_o$  [assumption (ii)]; this allows us to use the stability criteria of Ref. 6, which are discussed in the following part of this section. Since application of these criteria does not require a knowledge of the conversion properties of  $M$  at  $\omega_o \pm p$  and  $p$ , the analysis will be concerned exclusively with the behavior of  $M$  at  $\omega_o$  and dc.

### 2.1 Stability Criteria<sup>6</sup>

Let a one-terminal-pair network be constructed by connecting  $M$  to a dc source as shown in Fig. 3. The nonlinear impedance  $Z$  characterizing the terminal behavior at  $\omega_o$  of this network is a function of the amplitude  $I$  of  $i(t)$ . The form of this function depends upon the characteristics of the dc source. Of particular interest are the two cases arising when the dc source is: (i) an ideal current source with infinite internal impedance, or (ii) an ideal voltage source with zero internal impedance. It has been shown in Ref. 6 that a necessary and sufficient condition for the network  $M$  to be potentially unstable is

$$4R \frac{d(IR)}{dI} \leq \left( I \frac{dX}{dI} \right)^2 \quad (12)$$

in one of the above two cases.<sup>†</sup>

The network  $M$  imposes a set of nonlinear relations between its terminal voltages and currents at dc and  $\omega_o$ . Because of these relations, the impedance  $Z$  and the dc voltage  $V_o$  can be regarded as functions of  $I_o$  and  $I$ ,

$$\begin{aligned} R &= R(I_o, I), & X &= X(I_o, I) \\ V_o &= V_o(I_o, I). \end{aligned} \quad (13)$$

\* There are several ways of defining the gain of a mixer. A different definition will be used [see eq. (51)] in connection with the experiment described in Section VI, where the input signal generator will contain both  $\omega_o + p$  and  $\omega_o - p$ . However, the particular definition is immaterial to the analysis.

† For the network of Fig. 2 one can show that  $R > 0$  and  $\partial V_o / \partial I_o > 0$ ; if these two inequalities are not satisfied, the network would obviously be potentially unstable.

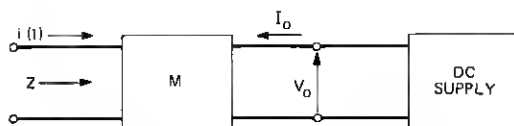


Fig. 3—Mixer connected to a dc bias supply.

These three functions completely describe the terminal behavior of  $M$ . If these functions are known, the derivatives appearing in inequality (12) can be evaluated as follows.

Note first that the two cases mentioned above correspond to the two conditions

$$I_o = \text{constant}, \quad (14)$$

$$V_o = \text{constant}. \quad (15)$$

Thus, in the former case, inequality (12) takes the form\*

$$4R \frac{\partial(IR)}{\partial I} \leq \left( I \frac{\partial X}{\partial I} \right)^2, \quad (16)$$

involving only partial derivatives of  $IR(I_o, I)$  and  $X(I_o, I)$  with respect to  $I$ .

In the latter case, differentiating eq. (15) we obtain

$$dI \frac{\partial V_o}{\partial I} + dI_o \frac{\partial V_o}{\partial I_o} = 0.$$

Thus,

$$\frac{dI_o}{dI} = - \frac{\partial V_o}{\partial I} \left( \frac{\partial V_o}{\partial I_o} \right)^{-1}.$$

Using this relation and the rule

$$\frac{d}{dI} = \frac{dI_o}{dI} \frac{\partial}{\partial I_o} + \frac{\partial}{\partial I},$$

we obtain for inequality (12)

$$4R \frac{\partial V_o}{\partial I_o} \left[ \frac{\partial V_o}{\partial I_o} \frac{\partial(IR)}{\partial I} - \frac{\partial V_o}{\partial I} \frac{\partial(IR)}{\partial I_o} \right] \leq \left[ I \frac{\partial X}{\partial I} \frac{\partial V_o}{\partial I_o} - I \frac{\partial X}{\partial I_o} \frac{\partial V_o}{\partial I} \right]^2. \quad (17)$$

Inequalities (16) and (17) will be useful in Section IV.

\* Throughout this paper  $\partial/\partial I_o$  (or  $\partial/\partial I$ ) is used to indicate differentiation with  $I$  (or  $I_o$ ) held constant.

In the following section we consider the behavior of  $M$  in a limiting case and find that a simple relation exists between  $Z$  and  $V_o$ ,  $I$ . Because of that relation, it is possible to ascertain stability directly from inequality (12), rather than using (16) and (17).

### III. ANALYSIS OF A LIMITING CASE

Analysis of the circuit of Fig. 2 is a considerable task, primarily because the current  $i_c$  through  $C(v_b)$  is not in general simply related to the total current  $i_b$  through the barrier. However, if the frequency  $\omega_o$  is sufficiently high, the current  $i_R$  absorbed by  $R(v_b)$  is much smaller than  $i_c$  for all  $t$ , and  $i_c$  is approximately equal to the alternating component of  $i_b$ .

$$i_c(t) \cong 2I \cos \omega_o t. \quad (18)$$

In the present section we study this case. Our main result is inequality (37).

If  $q$  and  $S$  denote the charge and elastance of the barrier capacitance respectively, then, since  $S = dv_b/dq$  and  $i_c = dq/dt$ , we can write

$$\frac{dS}{dt} = \frac{dS}{dv_b} \frac{dv_b}{dq} \frac{dq}{dt} = \frac{1}{2} \frac{d[S^2]}{dv_b} i_c. \quad (19)$$

From eq. (4) and the fact that  $S = C^{-1}(v_b)$ ,

$$S^2 = \frac{\phi - v_b}{(C_o \sqrt{\phi})^2}. \quad (20)$$

Taking the derivative and substituting in eq. (19) results in

$$\frac{dS}{dt} = - \frac{1}{2(C_o \sqrt{\phi})^2} i_c. \quad (21)$$

From this equation the alternative component of the elastance  $S(t)$  produced by the current  $i_c(t)$  of eq. (18) is determined. If  $S_o$  denotes the average value of  $S(t)$ , using eqs. (18) and (21),

$$S(t) = S_o + \frac{1}{(C_o \sqrt{\phi})^2} \left[ \frac{jI}{2\omega_o} e^{j\omega_o t} - \frac{jI}{2\omega_o} e^{-j\omega_o t} \right]. \quad (22)$$

Substituting this equation in eq. (20) results in

$$v_b(t) = \phi - (C_o \sqrt{\phi})^2 S_o^2 - \frac{I^2}{(C_o \sqrt{\phi})^2 2\omega_o^2} + \left( -j \frac{S_o}{\omega_o} I e^{j\omega_o t} + j \frac{S_o}{\omega_o} I e^{-j\omega_o t} + \dots \right), \quad (23)$$

where the dots indicate components at  $\pm 2\omega_o$ . The amplitudes of  $v_b(t)$  at dc and  $\omega_o$ , obtained from (23), are

$$V_{bo} = \phi - (C_o \sqrt{\phi})^2 S_o^2 - \frac{I}{(C_o \sqrt{\phi})^2 2\omega_o^2} \quad (24)$$

and

$$V = -j \frac{S_o I}{\omega_o} \quad (25)$$

Thus, the reactance  $X$  presented by the barrier capacitance at  $\omega_o$  can be written

$$X = -\frac{S_o}{\omega_o} \quad (26)$$

From eq. (24)

$$S_o = \frac{1}{C_o \sqrt{\phi}} \sqrt{\phi - V_{bo} - \frac{I^2}{(C_o \sqrt{\phi})^2 2\omega_o^2}} \quad (27)$$

Equations (26) and (27) specify the reactance  $X$  in terms of the two independent variables  $I$  and  $V_{bo}$ .

We now make the assumption that the diode is so operated that the power\* absorbed by  $R(v_b)$  at  $\omega_o$  is much smaller than the power dissipated in  $R_s$  at  $\omega_o$ . Because of this assumption, which is consistent with eq. (18), the impedance  $Z$  presented by the diode at  $\omega_o$  is simply  $R_s + jX$ . According to the preceding section the stability of the network of Fig. 2 can be ascertained from the behavior of  $R_s + jX$  using inequality (12), which reduces to

$$R_s \leq \frac{1}{2} I \left| \frac{dX}{dI} \right| \quad (28)$$

because  $R_s$  is independent of  $I$ . In order for the network of Fig. 2 to be potentially unstable, inequality (28) must be fulfilled under at least one of the two conditions (14) and (15).

Consider first condition (15), in which case  $V_{bo}$  can be assumed independent of  $I$  ( $V_{bo} \cong V_o$  because  $R_s I_o \cong 0$ ). From eqs. (26) and (27) one obtains

$$\frac{dX}{dI} = \frac{I}{2(C_o \sqrt{\phi})^4 \omega_o^3 S_o} \quad (29)$$

\* This power is  $\langle 2(Re)V_{b1}e^{j\omega_o t}i_R \rangle_{ave}$ ; it can be calculated to a first approximation using eqs. (5), (23), and (27).



Let  $S_M$  and  $S_m$  denote the maximum and minimum value of  $S(t)$ . From eq. (22) one can verify that  $I$  and  $S_o$  are related to  $S_M$  and  $S_m$  as follows:

$$S_o = \frac{S_M + S_m}{2} \quad (30)$$

$$I = \omega_o (C_o \sqrt{\phi})^2 \frac{S_M - S_m}{2}. \quad (31)$$

These relations and eq. (29) yield

$$I \frac{dX}{dI} = \frac{1}{4} \frac{S_M}{\omega_o} \frac{(1 - S_m/S_M)^2}{(1 + S_m/S_M)} \quad (32)$$

which is valid provided  $V_{bo}$  is independent of  $I$ .

Now consider condition (14) where  $V_{bo}$  is a function of  $I$ . In general, no simple relation exists between  $X$  and  $I$ ; however, in Appendix A it is shown that if for a given value of the ratio  $S_m/S_M$  the inequality

$$\left| \frac{qv_m}{kT} \right| \gg 1 \quad (33)$$

obtains [ $v_m$  is the minimum value of  $v_b(t)$ ], then condition (14) becomes equivalent to

$$S_m = \text{constant} \quad (34)$$

which leads to a simple relation between  $X$  and  $I$ . In fact, using eqs. (26), (30), and (31),  $X$  can be expressed in the form

$$X = -\frac{1}{\omega_o} \left[ S_m + \frac{I}{\omega_o (C_o \sqrt{\phi})^2} \right], \quad (35)$$

and  $X$  is linearly related to  $I$ . Further, using eqs. (31) and (35),

$$I \frac{dX}{dI} = -\frac{S_M - S_m}{2\omega_o}. \quad (36)$$

We now compare the two cases  $I_o = \text{constant}$  and  $V_o = \text{constant}$ , under the assumption that in the former case condition (33) is satisfied. From eqs. (32) and (36), for given  $S_M$ ,  $S_m$ , and  $\omega_o$ , eq. (36) gives a larger magnitude for  $I dX/dI$  than eq. (32). Since eq. (36) corresponds to the condition  $I_o = \text{constant}$ , we conclude that the network of Fig. 2 is potentially unstable only if inequality (28) is fulfilled in that case. From inequality (28) and eq. (36),

$$R_s \leq \frac{S_M - S_m}{4\omega_o}, \quad (37)$$

which gives the values of  $R_s$ ,  $S_M$ ,  $S_m$ , and  $\omega_o$  for which instability (and therefore high gain) is possible. If  $\omega_c$  denotes the cutoff frequency of the diode, so that  $\omega_c = S_M/R_s$ , and if  $S_m \ll S_M$ , then inequality (37) reduces to

$$\omega_c \geq 4\omega_o \quad (38)$$

which is eq. (1) for  $\eta = 4$ .

An understanding of the practical validity of inequality (37) is obtained by examining the restrictions in the above analysis. It has been assumed that the voltage across the barrier can be determined to a first approximation by neglecting the barrier resistance and also that the power absorbed at  $\omega_o$  by the barrier resistance is negligible compared with that dissipated in  $R_s$ . These assumptions are certainly satisfied if operation of the diode is restricted to a range of voltages  $v_b$  for which the barrier capacitance is predominant over the barrier resistance. However, this restriction is impractical because it would result in a mixer with very poor performance; for optimum performance the diode should be fully pumped; that is,  $v_b(t)$  should vary over the entire usable range of forward and reverse voltages. In the following section it is shown that inequality (37) is valid, approximately, even if the diode is fully pumped (in which case the restriction in question is not satisfied), *provided* the breakdown voltage  $V_B$  is sufficiently large. Thus, we can say that this requirement on  $V_B$  [which is in accord with the fact that inequality (37) has been derived under requirement (33)] is the main restriction on inequality (37).

#### IV. GENERAL CASE

According to eqs. (4) and (5), the voltage and current at the barrier are related through the nonlinear differential equation

$$\frac{C_o \sqrt{\phi}}{\sqrt{\phi} - v_b} \frac{dv_b}{dt} + i_s e^{(q/kT)v_b} - i_s - i_b = 0, \quad (39)$$

with  $i_b$  given by eqs. (6) and (7). This equation cannot in general be solved exactly, but an approximate solution can be obtained fairly simply to any degree of accuracy by the Euler method, as shown in Appendix B. Using that method,  $Z_b$ ,  $V_{bo}$ , and their partial derivatives with respect to  $I_o$  and  $I$  [these derivatives are needed to test the stability of M using inequalities (16) and (17)] have been calculated for various diode characteristics and terminal currents  $I_o$  and  $I$ . Table I shows

TABLE I—FOUR EXAMPLES OF THE BEHAVIOR OF M

	$\frac{i_s e^{(q\phi/kT)}}{C_o \omega_o \sqrt{\phi}} = 31.25$		$\frac{I_o + i_s}{C_o \omega_o \sqrt{\phi}} = 0.1$	
$ I /C_o \omega_o \sqrt{\phi}$	0.75	1.00	1.25	1.75
$V_{bo} - \phi$	-0.722	-1.116	-1.603	-2.857
$V_a - \phi$	-1.648	-2.660	-3.919	-7.180
$V_M - \phi$	-0.093	-0.089	-0.086	-0.082
$R_b \cdot \omega_o C_o \sqrt{\phi}$	0.110	0.101	0.096	0.090
$X_b \cdot \omega_o C_o \sqrt{\phi}$	-0.725	-0.904	-1.080	-1.432
$\partial V_{bo}/\partial I_o \cdot \omega_o C_o \sqrt{\phi}$	2.016	2.577	3.133	4.243
$\partial V_{bo}/\partial  I  \cdot \omega_o C_o \sqrt{\phi}$	-1.389	-1.762	-2.134	-2.878
$\partial( I/R_b )/\partial I_o \cdot \omega_o C_o \sqrt{\phi}$	0.576	0.774	0.966	1.345
$\partial( I/R_b )/\partial  I  \cdot \omega_o C_o \sqrt{\phi}$	0.0781	0.0763	0.0757	0.075
$ I  \partial X_b / \partial I_o \cdot \omega_o C_o \sqrt{\phi}$	1.403	1.784	2.166	2.937
$ I  \partial X_b / \partial  I  \cdot \omega_o C_o \sqrt{\phi}$	-0.542	-0.710	-0.881	-1.228

four examples where

$$\frac{i_s e^{(q\phi/kT)}}{C_o \omega_o \sqrt{\phi}} = 31.25, \quad \frac{I_o + i_s}{C_o \omega_o \sqrt{\phi}} = 0.1. \quad (40)$$

The four examples correspond to various values of the quantity  $I/C_o \omega_o \sqrt{\phi}$ , which represents the terminal current at  $\omega_o$ . The values  $v_m$  and  $v_M$  in Table I are the minimum and maximum value of  $v_b(t)$ . Note that according to eq. (20)  $S_M$  and  $S_m$  are related to  $v_M$  and  $v_m$  through the relations

$$S_m = \frac{\sqrt{\phi - v_M}}{C_o \sqrt{\phi}}, \quad S_M = \frac{\sqrt{\phi - v_m}}{C_o \sqrt{\phi}}. \quad (41)$$

One can verify that, in all the four cases of Table I, inequality (16) is violated for  $R_s = 0$ . Thus, in each case the circuit of Fig. 2 is potentially unstable provided the series resistance  $R_s$  is sufficiently small.

The values of  $R_s$  associated with potential instability can be derived as follows. In all the cases considered it has been found that if inequality (17) is fulfilled, then inequality (16) is also fulfilled; in other words, if  $Z$  satisfies inequality (12) under the condition  $V_a = \text{constant}$ , then it also satisfies inequality (12) under the condition  $I_a = \text{constant}$ . This property has already been found to be true in the limiting case discussed in the preceding section. It follows that for the network to be potentially unstable it is necessary (and, of course, sufficient) that  $Z = R_s + R_b + jX_b$  satisfy inequality (16). That is, it is necessary

that the expression

$$4(R_s + R_b) \left[ \left( \frac{\partial(IR_b)}{\partial I} \right) + R_s \right] - I^2 \left( \frac{\partial X_b}{\partial I} \right)^2 \quad (42)$$

be nonpositive. This requirement is equivalent to

$$R_s \leq R_{sc}, \quad (43)$$

where

$$R_{sc} = \frac{1}{2} \left[ \sqrt{I \left( \frac{\partial R_b}{\partial I} \right)^2 + I \left( \frac{\partial X_b}{\partial I} \right)^2} - \frac{\partial(IR_b)}{\partial I} - R_b \right]. \quad (44)$$

In fact, one can verify that expression (42) vanishes for  $R_s = R_{sc}$  and is negative for  $R_s \geq R_{sc}$ . According to inequality (43),  $R_{sc}$  is the largest series resistance for which the network of Fig. 2 is potentially unstable.

Figure 4 shows several curves of  $R_{sc}$  versus  $|v_m - \phi|$  calculated [using Eq. (44)] for different values of  $I_o/C_o\omega_o\sqrt{\phi}$  and for  $i_s[\exp(q\phi/kT)]/\omega_o C_o\sqrt{\phi} = 31.25$ . The four points indicated on the curve relative to  $I_o/C_o\omega_o\sqrt{\phi} = 0.1$  correspond to the four cases of Table I, as one can

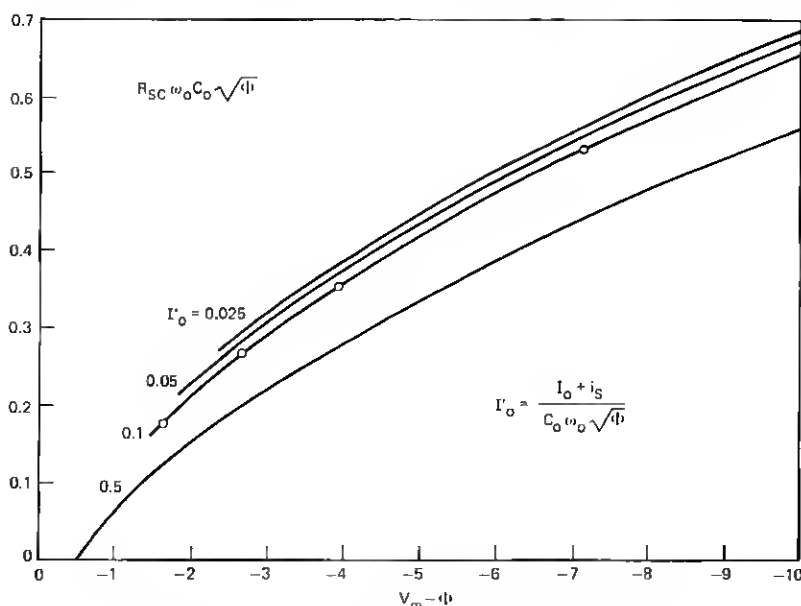


Fig. 4—Behavior of  $R_{sc}$  as a function of  $v_m$  for different values of  $I_o$ .

verify using eq. (44). We see from Fig. 4 that for given diode characteristics and for a given  $\omega_o$ , the behavior of  $R_{sc}$  as a function of  $I_o$  and  $|v_m - \phi|$  has the following characteristics. For a given  $I_o$ ,  $R_{sc}$  increases monotonically with  $|v_m - \phi|$ . Since  $v_m$  cannot exceed the breakdown voltage,  $V_B$ , the largest value of  $R_{sc}$  for a given  $I_o$  occurs when  $v_m = V_B$ . For a given  $v_m$ ,  $R_{sc}$  increases with decreasing  $I_o$  and approaches a finite limit for  $I_o \rightarrow 0$ . Figure 4 shows that, if  $|V_m - \phi| > 2$  volts,  $R_{sc}$  is little affected by the value of  $I_o$  for  $I_o/\omega_o C_o \sqrt{\phi} < 0.1$ , approximately.

Now from the discussion of the limiting case of Section III, we know that, if conditions (18) and (33) are satisfied, then according to inequality (37)

$$\frac{R_{sc}\omega_o}{S_M - S_m} = \frac{1}{4}. \quad (45)$$

Condition (18) can be assumed to be fulfilled when  $I_o/\omega_o C_o \sqrt{\phi}$  is sufficiently small, in which case the quantity  $R_{sc}\omega_o/(S_M - S_m)$  is expected to approach 1/4 for large  $|v_m - \phi|$ . It is interesting to compare this asymptotic behavior of  $R_{sc}\omega_o/(S_M - S_m)$  with the behavior corresponding to the curves of Fig. 4. Figure 5 shows  $R_{sc}\omega_o/(S_M - S_m)$  plotted as a function of  $|v_m - \phi|$  for the four cases corresponding to

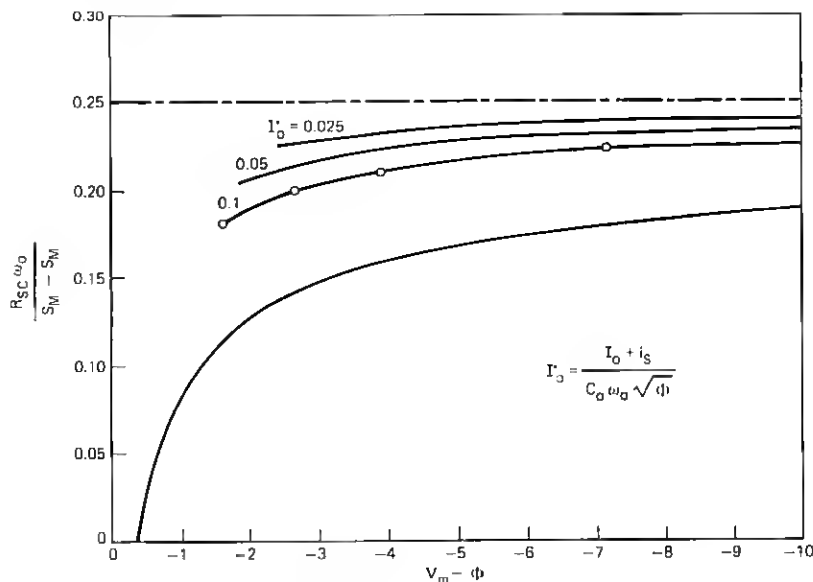


Fig. 5—Behavior of  $R_{sc}\omega_o/(S_M - S_m)$ .

the curves of Fig. 4. One sees that  $R_{sc}\omega_o/(S_M - S_m)$  never exceeds 1/4, and that if  $I_o/\omega_o C_o \sqrt{\phi}$  is sufficiently small,  $R_{sc}\omega_o/(S_M - S_m)$  approaches 1/4 for large  $|v_m - \phi|$ , as expected. The four points indicated in Fig. 5 correspond to the cases of Table I.

So far, the quantity  $i_s[\exp(q\phi/kT)]/\omega_o C_o \sqrt{\phi}$  has been assumed to be 31.25; Fig. 6 shows how  $R_{sc}/\omega_o C_o \sqrt{\phi}$  is affected if  $i_s[\exp(q\phi/kT)]/\omega_o C_o \sqrt{\phi}$  is changed from 31.25 to 1.08. The two curves of Fig. 6 have been calculated for  $I_o/\omega_o C_o \sqrt{\phi} = 0.1$ . It is evident that  $R_{sc}/\omega_o C_o \sqrt{\phi}$  does not depend critically on the value of  $i_s/\omega_o C_o \sqrt{\phi}$ . The four points indicated in Fig. 6 correspond to the four cases of Table I.

#### 4.1 Region of Instability for Typical Diode Characteristics

Typically, the breakdown voltage is sufficiently large so that one can assume  $\phi - v_m > 2$  volts. For such values of  $\phi - v_m$  one can verify from Fig. 4 that  $R_{sc}\omega_o C_o \sqrt{\phi}/\sqrt{\phi - v_m} > 0.16$  if  $I_o/\omega_o C_o \sqrt{\phi} < 0.5$ . Thus, one can assume for typical diodes

$$0.16 < \frac{R_{sc}\omega_o C_o \sqrt{\phi}}{\sqrt{\phi - v_m}} < 0.25. \quad (46)$$

In the introduction, the range of pump frequencies associated with potentially unstable behavior was expressed in terms of the parameter  $\eta$ . A comparison of inequalities (1) and (43) shows that this parameter is

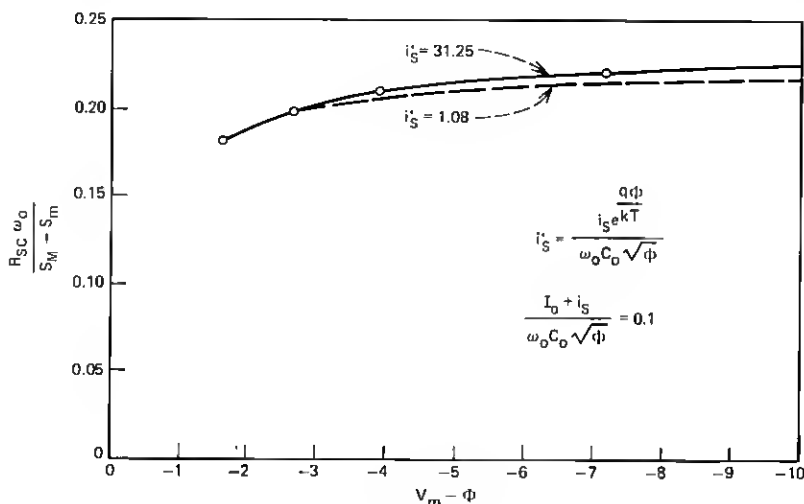


Fig. 6—Effect of  $i_s$ .

related to  $R_{sc}$  as follows:

$$\eta = \frac{\omega_c R_s}{\omega_o R_{sc}} = \frac{S_M}{\omega_o R_{sc}} = \frac{\sqrt{\phi - v_m}}{R_{sc} \omega_o C_o \sqrt{\phi}}. \quad (47)$$

Thus, according to inequality (46),  $\eta$  typically lies between 4 and 6.25, as was stated in the introduction.

#### V. EFFECT OF $R_s$ ON NOISE PERFORMANCE AT HIGH GAIN

The limit imposed by  $R_s$  on the optimum noise performance obtainable at high gain is now derived, when the diode is used as a down-converter and is terminated with equal impedances at  $\omega_o \pm p$ . The effect of  $R_s$  on noise can be separated into two parts: one is the effect at  $\omega_o \pm p$ , and the other at the output frequency  $p$ . We will see that if the diode is operated at high gain, the effect at  $\omega_o \pm p$  is minimized when the diode is terminated with a very high impedance at  $\omega = p$ . Under this condition the effect of  $R_s$  at  $\omega = p$  vanishes and can therefore be ignored.

Consider the effect of  $R_s$  at  $\omega_o \pm p$ . If  $R_a$  denotes the real part of the equal terminations at  $\omega_o + p$  and  $\omega_o - p$ , the real part of the total impedance terminating the barrier at  $\omega_o \pm p$  is  $R_{at} = R_a + R_s$ . Now inequality (43) implies that high gain is possible only if  $R_{at} \leq R_{sc}$ , that is, only if

$$R_a \leq R_{sc} - R_s. \quad (48)$$

Furthermore, when the resistance  $R_{at}$  equals  $R_{sc}$  (i.e., when  $R_a = R_{sc} - R_s$ ), one can show that high gain requires a very high output impedance in all cases of Section IV. (This is a direct consequence of the fact that, when the resistance seen by the barrier at  $\omega_o$  equals  $R_{sc}$ , instability may arise only if  $I_o = \text{constant}$ ; that is, only if the diode is biased by a dc supply with infinite internal impedance.)

Now the ratio of the thermal noise power available (at  $\omega_o \pm p$ ) at the barrier to that available at the terminals of the diode is  $R_{at}/R_a$ . This represents the impairment caused by the presence of  $R_s$  at  $\omega_o \pm p$  on the noise performance of the diode as a down-converter. According to inequality (48), this impairment is minimized when  $R_a$  equals  $R_{sc} - R_s$ , in which case a very high termination is required at  $\omega = p$ . Thus, since the effect of  $R_s$  at  $\omega = p$  vanishes, and at  $\omega = \omega_o \pm p$  is given by the ratio  $R_{sc}/(R_{sc} - R_s)$ , the noise figure can be written

$$F_m = \frac{R_{sc}}{R_{sc} - R_s} F, \quad (49)$$

where  $F$  is the noise figure obtainable at high gain in the ideal case  $R_s = 0$ , when the termination at  $\omega = p$  is a very high impedance. Using eq. (47) this relation can be rewritten

$$F_m = \left(1 - \eta \frac{\omega_o}{\omega_c}\right)^{-1} F \quad (50)$$

from which one obtains inequality (3).

## VI. EXPERIMENTAL RESULTS\*

Experimental data have been obtained at a pump frequency of 1.25 GHz using a GaAs Schottky barrier diode,<sup>†</sup> having  $R_s \cong 4$  ohms,  $C_o \cong 1.2$  pF, and  $V_B \cong 10$  volts. This diode was mounted in a circuit designed to produce at the barrier very high terminating impedances at  $2\omega_o$ ,  $3\omega_o$ , and  $4\omega_o$ . The structure is shown in Fig. 7 with the cover plate removed; the diode is inserted between two resonators both of which operate in the TEM mode. One resonator is connected to a 50-ohm coaxial line and consists of a main line with two series-resonant circuits connected in shunt. The purpose of the two series-resonant circuits is to short out transmission at  $2\omega_o$  and  $4\omega_o$  between the 50-ohm coaxial line and the diode. Each circuit consists of a line element connected to the main line at one end, with a lumped capacitance between the other end of each line element and ground.

The main line has the diode chuck soldered at one end; the other end is open-circuited. The electrical distance between the open-circuited end and the connection point at which the coaxial line is attached is a quarter of a wavelength at  $3\omega_o$ . The connection point of the coaxial line is therefore also short-circuited at  $3\omega_o$ . The distance from this connection point to the diode and the dimensions of the other resonator were chosen so as to open-circuit the barrier of the diode at  $2\omega_o$ ,  $3\omega_o$ , and  $4\omega_o$ , using the following procedure. Initial dimensions for the two resonators were obtained experimentally by adjusting one resonator at the time (the other resonator and the diode being removed). The L-shaped resonator was adjusted so as to obtain two resonances at  $2\omega_o$  and  $4\omega_o$ . The other resonator was adjusted (with the 50-ohm line terminated in 50 ohms) for a resonance at  $3\omega_o$ . Then an empty package identical to that of the diode used in this experiment was mounted between the two resonators as shown in Fig. 7, and the resonant frequencies of this circuit were measured by coupling the circuit

\* The experiment described in this section was carried out by S. Michael of Bell Laboratories.

<sup>†</sup> Supplied by J. C. Irvin of Bell Laboratories.



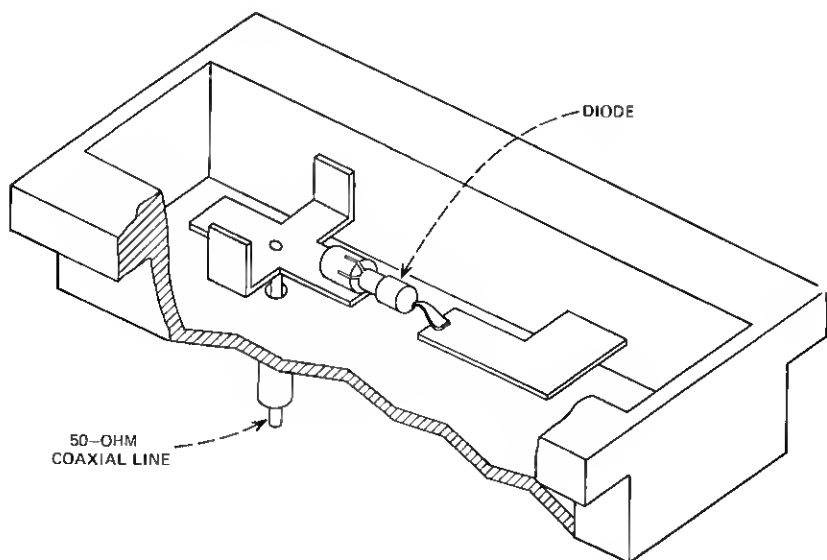


Fig. 7—Circuit used to open-circuit the barrier of the diode at  $2\omega_o$ ,  $3\omega_o$ , and  $4\omega_o$ .

to a load and a generator by means of two loosely coupled capacitive probes. The resonant frequencies were found to be appreciably different from  $2\omega_o$ ,  $3\omega_o$ , and  $4\omega_o$  as expected, because of the coupling between the two resonators resulting from the case capacitance of the package. The dimensions of the two resonators were then adjusted to give the desired resonances at  $2\omega_o$ ,  $3\omega_o$ , and  $4\omega_o$  and the empty package was finally replaced with the actual diode.

In order to separate the pump frequency  $\omega_o$  from dc, the 50-ohm coaxial line of the circuit of Fig. 7 was connected to one of the three arms of a monitor tee consisting of a main line shunted by an auxiliary line. The two lines are provided with a capacitor and an inductor connected in series to their central conductors to block signals in the vicinity of dc and  $\omega_o$ , respectively. The other two arms of the monitor tee are connected to an output matching network and a tuner, as shown schematically in Fig. 8.

Figure 9 shows a block diagram of the apparatus used to measure the noise characteristics of the mixer of Fig. 8. The noise source is the AIL type 70 Hot-Cold Body Standard Noise Generator [consisting of two terminations, one immersed in liquid nitrogen ( $77.3^\circ\text{K}$ ) and the other mounted in a temperature-controlled oven ( $373.2^\circ\text{K}$ )]. The pump is connected to a narrowband filter. This filter consists of four identical

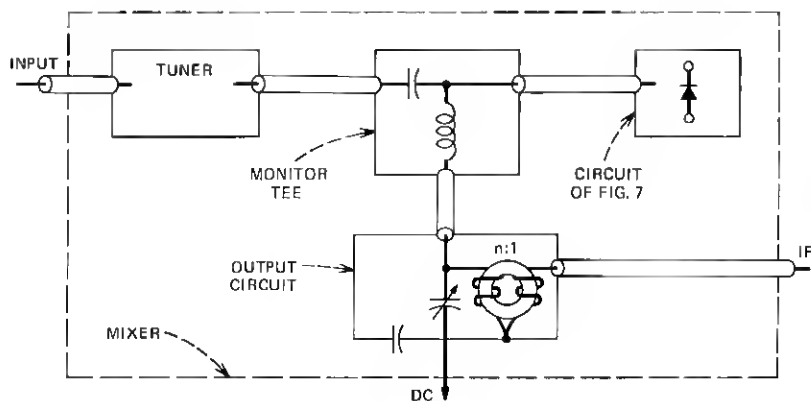
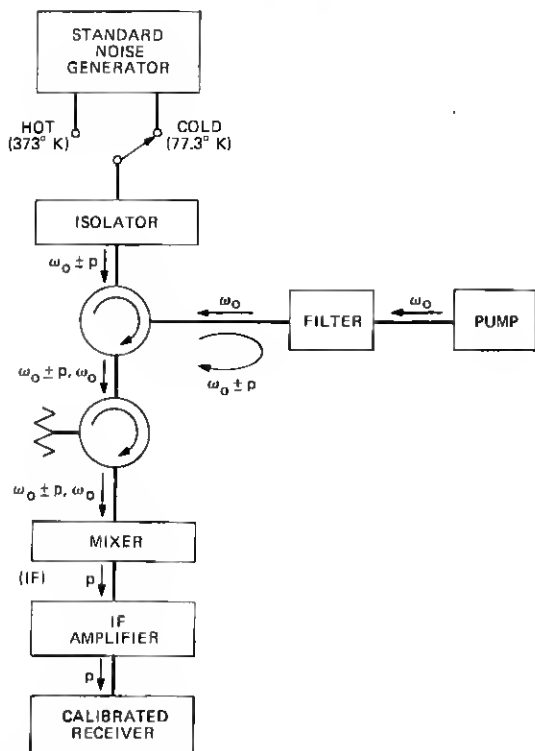


Fig. 8—Mixer.

Fig. 9—Block diagram of apparatus used to test the mixer of Fig. 8;  $\omega_o = 1.25$  GHz;  $p = 2$  MHz.

cavity resonators tuned at  $\omega_0$  and separated by transmission lines  $\lambda/4$  long at  $\omega_0$ ; the attenuation at  $\omega_0$  is approximately 10 dB and, at  $\omega_0 \pm p$  for  $p \geq 2$  MHz, is greater than 55 dB; its fractional bandwidth is approximately 0.07 percent. The noise source is buffered by an isolator providing more than 22 dB isolation at  $\omega_0 \pm p$ . The noise components originating from the source at  $\omega_0 \pm p$  ( $p \geq 2$  MHz) enter the first circulator and are directed into the pump filter which reflects them back to the circulator where they are directed into a second circulator. The second circulator, which provides 22 dB of isolation at  $\omega_0 \pm p$ , is followed by the mixer, the circuit of which is shown in Figs. 8 and 9. The noise power entering the mixer at  $\omega_0 \pm p$  is converted to the output (IF) frequency  $p \cong 2$  MHz. The converted power is then amplified and finally measured with a narrowband receiver. The IF amplifier system consists of a calibrated variable attenuator followed by an amplifier, the noise figure of which was optimized at 0.19 dB for  $p = 2$  MHz. The purpose of the variable attenuator is to vary the noise figure  $F_i$  of the IF amplifier.

The double-sideband noise figure of the mixer-amplifier combination is given by the familiar relation

$$F_r = F_m + \frac{F_i - 1}{G_m}, \quad (51)$$

where  $F_m$  is the double-sideband noise figure of the mixer\* and  $G_m$  is its gain. Note that  $G_m$  is not the conversion gain from  $\omega_0 + p$  to  $p$ , or from  $\omega_0 - p$  to  $p$ , but is the sum of the two. That is, if  $G_{\alpha\beta}$  and  $G_{\gamma\beta}$  denote these two gains,  $G_m = G_{\alpha\beta} + G_{\gamma\beta}$  (thus  $G_m \cong 2G_{\alpha\beta}$  because  $G_{\alpha\beta} \cong G_{\gamma\beta}$ ). Measurement of  $F_r$  consists essentially of determining two quantities: (i) the ratio  $Y = P_2/P_1$ , where  $P_2$  and  $P_1$  are the power outputs of the IF amplifier corresponding to the two temperatures  $T_2 = 373.2^\circ\text{K}$  and  $T_1 = 77.3^\circ\text{K}$  and (ii) the insertion loss  $\ell$  of the circuit connected between the source and the mixer; this loss is less than 1 dB and can be measured very accurately. The noise figure is given by the well-known formula  $F_r = [1 + (T_2 - T_1)Y/290(Y - 1)]\ell$ . The accuracy of measurement of  $F_r$  is limited primarily by the accuracy of the two temperatures  $T_2$  and  $T_1$ ; the estimated error for  $F_r$  is less than 0.1 dB.

According to eq. (51),  $F_m$  and  $G_m$  can be determined indirectly by measuring the effects of  $F_r$  of varying the IF noise figure  $F_i$ . Figure 10 shows a plot of  $F_r$  versus  $F_i$  which was obtained after adjusting the

\*  $F_m$  is the ratio of the total noise power output of the mixer to that portion of this power originating from the terminations of the mixer at  $\omega_0 + p$  and  $\omega_0 - p$ , assuming these terminations are at 290°K.

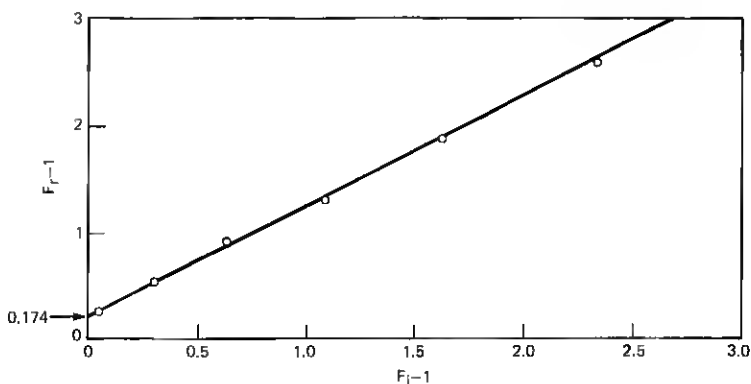


Fig. 10—Behavior of  $F_r$  versus  $F_i$ ; mixer optimized for  $F_i = 0.19$  dB.

mixer for minimum  $F_r$  under the particular condition  $F_i = 0.19$  dB. According to eq. (51) the slope of this curve is the mixer conversion loss  $L_m = 1/G_m$  and the point corresponding to  $F_i = 1$  is  $F_m$ . This curve tells us that for a gain of 0.948 the lowest noise figure obtainable from the mixer is 1.174.

Figure 11 shows a plot of the minimum noise figure of the mixer versus its conversion loss. From this plot we can derive the lowest  $F_r$  obtainable for a given  $F_i$  as follows. From eq. (51), after replacing  $1/G_m$  with  $L_m$  and differentiating, we get

$$\frac{dF_r}{dL_m} = \frac{dF_m}{dL_m} + (F_i - 1). \quad (52)$$

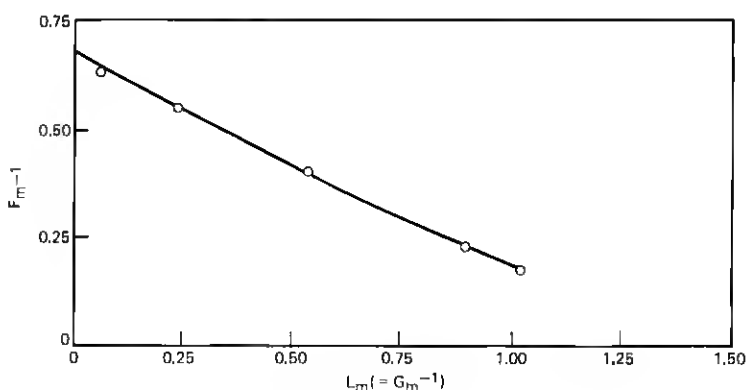


Fig. 11—Behavior of  $F_m$  versus  $L_m = G_m^{-1}$ .

Because the curve of Fig. 11 has  $d^2F_m/d^2L_m > 0$ , it follows that a point on the curve minimizes  $F_r$  if it satisfies  $dF_r/dL_m = 0$  which, from eq. (52), results in

$$F_i = 1 - \frac{dF_m}{dL_m}. \quad (53)$$

Using this equation, we obtain from Fig. 11 the curve of Fig. 12, which shows the relation between  $F_i$  and the optimum value of  $L_m$ . One sees that it is desirable to have  $L_m < 1$  (i.e., gain greater than unity) when

$$F_i > 1.37 (\sim 1.4 \text{ dB}). \quad (54)$$

## VII. CONCLUSIONS

It has been shown that a Schottky barrier diode is capable of arbitrarily high conversion gain as a mixer provided  $\omega_o < \omega_c/\eta$ , where  $\eta$  is a parameter typically less than 6.25, but always greater than 4. If  $\omega_o \ll \omega_c/\eta$ , then according to inequality (3), the ultimate double-sideband noise figure at high gain should be very close to unity. A fortiori, the ultimate noise figure at low gain should be excellent, if  $\omega_o \ll \omega_c/\eta$ . These conclusions are corroborated by the experimental results. Although the experimental result obtained is very good ( $F_m$ , 0.7 to 2.3 dB), it does not achieve the theoretical limit, nor was it expected to; practical limitations such as input circuit losses, estimated to exceed 0.4 dB for  $G_m \gg 1$ , confine the experimental noise figure for  $G_m \gg 1$  to values appreciably higher than the theoretical limit.

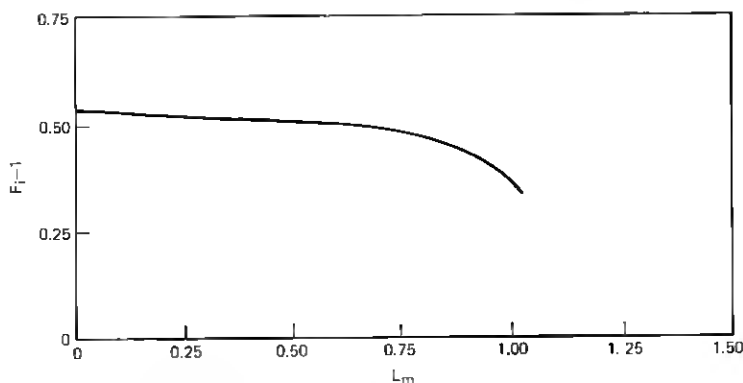


Fig. 12--Relation between  $F_i$  and the optimum value of  $L_m$ .

## VIII. ACKNOWLEDGMENTS

The author is indebted to D. C. Hogg for his help and guidance in the preparation of this paper. Thanks are also expressed to C. L. Ruthroff for a number of useful discussions during the course of the work.

## APPENDIX A

In this appendix, we analyze the condition  $I_o = \text{constant}$  for the case of  $|qv_m/kT| \gg 1$  of Section III. The average current through the barrier resistance is

$$I_o = \langle i_e e^{(q/kT)v_b(t)} \rangle_{\text{ave}} - i_s. \quad (55)$$

Thus, if a perturbation  $\delta I$  is applied to the diode current at  $\omega_o$ , while  $I_o$  is held constant, from eq. (55) the resulting perturbation of  $v_b(t)$  must satisfy the condition

$$\langle e^{+(q/kT)v_b(t)} \delta v_b(t) \rangle_{\text{ave}} = 0. \quad (56)$$

According to eq. (20) this condition can be rewritten in the form

$$\langle e^{-BS^2(t)} S(t) \delta S(t) \rangle_{\text{ave}} = 0, \quad (57)$$

where

$$B = \frac{q}{kT} (C_o \sqrt{\phi})^2. \quad (58)$$

Note that  $S(t)$  is completely specified by its maximum and minimum value  $S_M$  and  $S_m$ . We will presently show that if for a given value of the ratio

$$r = \frac{S_m}{S_M} \quad (59)$$

we let  $S_M \rightarrow \infty$  then the time function  $\exp[-BS^2(t)]$  over the interval  $-T/2 \leq t \leq T/2$  approaches an impulse located at  $t = 0$ . Thus, if  $A$  denotes the area of this impulse, we will show that for a given value of  $r$  we can write:

$$\exp[-BS^2(t)] \rightarrow A u_o(t) \quad (|t| \leq T_o/2) \quad (60)$$

for  $S_M \rightarrow \infty$ , where  $u_o(t)$  denotes the unit impulse. From this result and the fact that  $S(0) = S_m$  we find that for  $S_M \rightarrow \infty$  Eq. (56) becomes

$$S(0) \delta S(0) = S_m \delta S_m = 0. \quad (61)$$

We can thus say that for  $S_M \rightarrow \infty$ , condition (14) is equivalent to condition (34). We now show that relation (60) is true for  $S_M \rightarrow \infty$ .

Let  $\epsilon$  be an arbitrarily small positive quantity. We have to show that for a given value of  $r$

$$\lim_{S_M \rightarrow \infty} \frac{\int_{\epsilon}^{T/2} \exp[-BS^2(t)] dt}{\int_0^{T/2} \exp[-BS^2(t)] dt} = 0. \quad (62)$$

First, note that  $S(t)$  can be written

$$S(t) = S_M \left[ \frac{1+r}{2} - \frac{1-r}{2} \cos \omega_o t \right]. \quad (63)$$

Using this expression one can verify that  $\exp[-BS^2(t)] < \exp[-BS^2(\epsilon)]$  for  $\epsilon < t \leq T/2$ . Thus,

$$\int_{\epsilon}^{T/2} \exp[-BS^2(t)] dt < \frac{T}{2} \exp[-BS^2(\epsilon)]. \quad (64)$$

Furthermore, it can be shown that if  $S_M B$  is sufficiently large then

$$\int_0^{T/2} \exp[-BS^2(t)] dt > \frac{T}{2} \frac{e^{-r^2 S_M^2 B}}{\sqrt{2\pi r(1-r)BS_M^2}}. \quad (65)$$

In fact, since  $\cos \omega_o t > 1 - (\omega_o t)^2/2$ , from eq. (63)

$$\begin{aligned} S^2(t) &< S_M^2 \left[ r + \frac{(1-r)}{4} (\omega_o t)^2 \right]^2 \\ &= S_M^2 \left[ r^2 + \frac{(1-r)r}{2} (\omega_o t)^2 + \frac{(1-r)^2}{16} (\omega_o t)^4 \right]. \end{aligned}$$

This inequality allows one to write

$$\begin{aligned} &\int_0^{T/2} \exp[-BS^2(t)] dt \\ &> \frac{T}{2\pi} \frac{e^{-r^2 S_M^2 B}}{\sqrt{r(1-r)BS_M^2/2}} \int_0^{\gamma \sqrt{r(1-r)BS_M^2/2}} \exp[-u^2 - \gamma u^4] du, \end{aligned}$$

where  $\gamma = (4BS_M^2 r^2)^{-1}$ . If  $BS_M$  is sufficiently large, so that  $\gamma \ll 1$ , then from this inequality we obtain (65).

From inequalities (64) and (65) we obtain

$$\frac{\int_{\epsilon}^{T/2} \exp[-BS^2(t)] dt}{\int_0^{T/2} \exp[-BS^2(t)] dt} < \sqrt{2\pi BS_M^2(1-r)r} e^{-B[S^2(\epsilon) - r^2 S_M^2]}. \quad (66)$$

According to Eq. (63) we can write

$$S^2(\epsilon) - r^2 S_M^2 = S_M^2 \epsilon_r(\epsilon), \quad (67)$$

where  $\epsilon_r(\epsilon)$  is a positive quantity which depends upon  $r$  and  $\epsilon$ , but is independent of  $S_M$ . From eq. (67) and inequality (66) we therefore conclude that eq. (62) is true. Note that, since  $\epsilon$  is a small quantity, from eqs. (63) and (67) we can write

$$\epsilon_r(\epsilon) \cong \frac{(1-r)r\epsilon^2}{2}.$$

Thus,

$$\frac{\int_{\epsilon}^{T/2} \exp[-BS^2(t)] dt}{\int_0^{T/2} \exp[-BS^2(t)] dt} \ll 1$$

provided

$$BS_M^2 \gg \frac{2}{(1-r)r\epsilon^2}.$$

#### APPENDIX B

Figure 13 shows a network consisting of a variable elastance, a variable resistance, and a constant resistance  $R'_s$ . Assume that the current  $i'_R$  through the variable resistance is related to the voltage  $v'_b$  as follows:

$$i'_R = i'_s e^{qv_b'/kT} \quad (68)$$

and that the variation of the elastance  $S'$  is characterized by the relation

$$S' = \sqrt{-v'_b}. \quad (69)$$

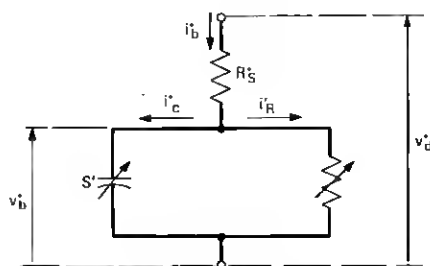


Fig. 13—Network representing a Schottky barrier diode.



Let the terminal current  $i'_b$  of this network be a periodic function of  $\tau$  with period  $2\pi$ , so that

$$i'_b = i'_b(\tau) = i'_b(\tau + 2\pi). \quad (70)$$

Then,  $i'_b$  and  $v'_b$  are related through the differential equation

$$\frac{1}{\sqrt{-v'_b}} \frac{dv'_b}{d\tau} + i'_b e^{(a/kT)v'_b} - i'_b = 0. \quad (71)$$

Furthermore, one sees from Fig. 13 that the terminal voltage  $v'_d$  can be written

$$v'_d = i'_b R'_s + v'_b. \quad (72)$$

A Schottky barrier diode can always be represented by such a network. In fact, if we assume

$$\begin{aligned} \tau &= \omega_o t \\ v'_b &= v_b - \phi, & v'_d &= v_d - \phi + i_s R_s \\ i'_b &= \frac{i_s e^{(a\phi/kT)}}{C_o \omega_o \sqrt{\phi}} \\ i'_b &= \frac{i_b}{C_o \omega_o \sqrt{\phi}} + \frac{i_s}{C_o \omega_o \sqrt{\phi}} \\ R'_s &= R_s \omega_o C_o \sqrt{\phi} \end{aligned} \quad (73)$$

and substitute these relations in eqs. (71) and (72), we obtain eq. (39) and the relation  $v_d = i_b R_s + v_b$ . Thus, the two networks of Figs. 11 and 13 are equivalent.

### Analysis

Let us now assume for  $i'_b$  the form

$$i'_b = i'_b(\tau) = \begin{cases} C, & \text{for } \tau \leq 0 \\ I'_s + 2I' \cos \tau, & \text{for } \tau > 0 \end{cases} \quad (74)$$

where  $C$  is a constant. Since then  $i'_b = i'_R = C$  for  $\tau \leq 0$ , from eq. (68):

$$v'_b = v'_b(\tau) = \frac{kT}{q} (\ln C - \ln i'_s) \quad \text{for } \tau \leq 0. \quad (75)$$

Since for  $\tau \geq 0$ ,  $i'_b(\tau)$  is periodic, it is reasonable to expect that for  $\tau$  sufficiently large,  $v'_b(\tau)$  will also be periodic. Let us suppose there is a

positive integer  $N$  such that one can write to a good approximation

$$v'_b(2\pi N + 2\pi) \cong v'_b(2\pi N), \quad (76)$$

so that  $v'_b(\tau)$  can be assumed to be periodic for  $\tau \geq 2\pi N$ ,

$$v'_b(\tau + 2\pi) \cong v'_b(\tau), \quad \text{for } \tau \geq 2\pi N. \quad (77)$$

Then we can write

$$v'_b(\tau) \cong v'_{b0} + 2(\text{Re})(v'_{b1}e^{i\tau} + \dots) \quad \text{for } \tau \geq 2\pi N, \quad (78)$$

where the dots indicate the components of order 2, 3, etc., and

$$V'_{b0} = \frac{1}{2\pi} \int_{2\pi N}^{2\pi N + 2\pi} v'_b(\tau) d\tau \quad (79)$$

and

$$V'_{b1} = \frac{1}{2\pi} \int_{2\pi N}^{2\pi N + 2\pi} v'_b(\tau) e^{-i\tau} dt. \quad (80)$$

Thus, if we can determine the behavior of  $v'_b(\tau)$  over the interval  $2\pi N \leq \tau \leq 2\pi(N + 1)$ , where  $N$  is the smallest positive integer for which condition (76) can be assumed to be fulfilled, then the coefficients  $V'_{b0}$  and  $V'_{b1}$  can be readily determined by these two relations.

For given values of  $C$ ,  $I'_o$  and  $I'$ , an approximate solution to eq. (71) can be obtained using the Euler method. This method requires that the continuous variable  $\tau$  be replaced by the discrete variable  $n\tau_o$  ( $n = 0, \pm 1, \pm 2$ , etc.). Since in eq. (74) for  $\tau \geq 0$   $i'_b(\tau)$  has period  $2\pi$ , it is convenient to assume for the step size  $\tau_o$  an exact submultiple of  $2\pi$ ,

$$\tau_o = \frac{2\pi}{P} \quad (81)$$

where  $P$  is a positive integer. In the Euler method of solution, eq. (71) is replaced by the difference equation

$$v'_{n+1} = v'_n + D_n \tau_o, \quad (82)$$

where  $v'_n$  is the approximation to  $v'_b$  for  $\tau = n\tau_o$ , and  $D_n$  is the approximation to the derivative  $dv'_b/d\tau$  for  $\tau = n\tau_o$ . From eq. (71)

$$D_n = \sqrt{-v_n'} (i'_n - i'_o e^{(q/kT)\tau_n'}) \quad (83)$$

where

$$i'_n = \begin{cases} I'_o + 2I' \cos(n\tau_o), & \text{for } n > 0 \\ C, & \text{for } n = 0 \end{cases} \quad (84)$$

Equations (82) to (84) show that the values of  $v_o$ ,  $v_1$ ,  $v_2$ , etc. can be calculated sequentially, starting with

$$D_o = 0, \quad v'_o = \frac{kT}{q} \ln \left( \frac{i'_o}{i'_s} \right), \quad (85)$$

then calculating  $v'_1$  and  $D_1$ , and so on. Let  $N$  be the smallest integer for which the condition

$$v'_{P(N+1)} = v'_{PN} \quad (86)$$

is satisfied. Then, according to eqs. (79) and (80) the desired approximations to  $V'_o$  and  $V'$  are

$$V'_o = \frac{1}{P} \sum_{K=0}^{P-1} v'_{K+PN} \quad (87)$$

$$V' = \frac{1}{P} \sum_{K=0}^{P-1} v'_{K+PN} e^{-i\tau_{K+PN}}.$$

Table I\* and Figs. 4 to 6 have been calculated choosing as initial condition

$$i'_o = C = I'_o + 2I'. \quad (88)$$

The value of  $N$  depends on the value of  $I'_o$ . One can show that  $N \rightarrow \infty$  for  $I'_o \rightarrow 0$ . Thus, the above method is not suitable if  $I'_o$  is too small. However, for the values of  $I'_o$  that are of practical interest,  $N$  is typically a small integer; for instance,  $N \leq 4$  in all the cases in Table I.

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\* Note that according to eqs. (73) the various expressions of Table I have the following significance:  $(I_o + i_s)/C_o\omega_o\sqrt{\phi} = I_o'$ ;  $I/C_o\omega_o\sqrt{\phi} = I'$ ;  $V_{bo} - \phi = V_{bo}'$ ;  $V_m - \phi$  and  $V_M - \phi$  are the minimum and maximum value of  $v_b(\tau)$ ;  $Z_b\omega_o C_o\sqrt{\phi} = Z_b' = V_b'/I'$ ; etc.

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